

Inter-Species Collisionless Energy transfer by drift wave-zonal flow Turbulence

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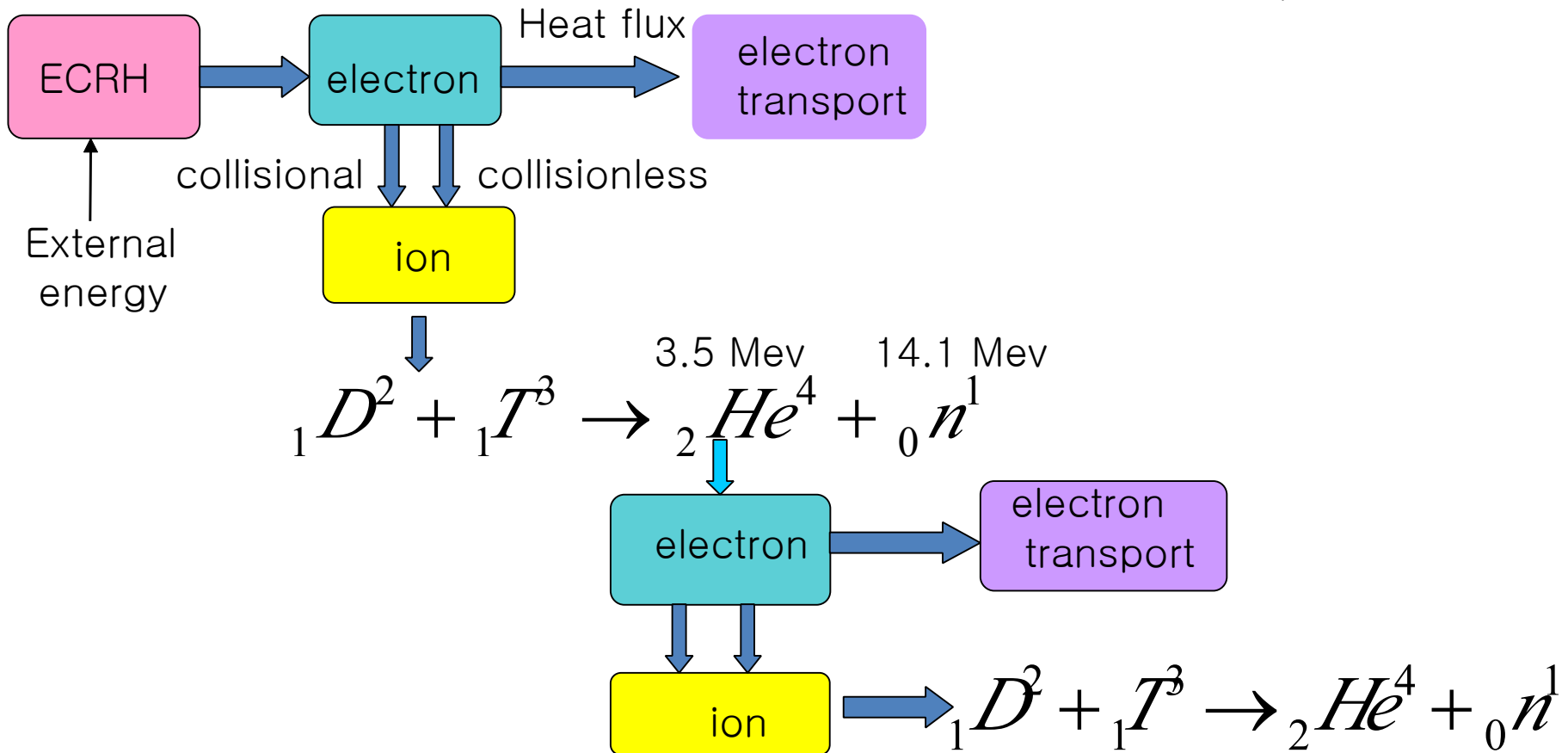
Outline

- *We reconsider the classic problem of “**turbulent heating**” and the inter-species transfer of energy in drift wave turbulence*
- Motivation: **Transfer** vs **Transport** → Roles in energy budget
 - Consider
 - ┌ Net volumetric heating → Does turbulence heat a given volume of plasma?
 - └ Electron → ion collisionless energy transfer channels
- Calculate and Estimate Energy Transfer Channels
 - ┌ Electron cooling : quasilinear
 - └ Ion heating : quasilinear, nonlinear, Ion Pol & Dia
- Implication for ITER/Result and discussion
 - ┌ Turbulent vs collisional transfer
 - └ Turbulent transport vs Turbulent transfer
- **Special Topics ongoing:**
 - ┌ Nonlinear electron cooling in CTEM
 - └ Origin of temperature profile "stiffness"
- Conclusion

Background

- ITER: collisionless, electron heated plasma

- Two stages in energy flow $\frac{3}{2}n\frac{\partial T_e}{\partial t} + \nabla \cdot \mathcal{Q}_e = \langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}}_e \rangle - n\nu_{e,i} \frac{m_e}{m_i} (T_e - T_i) + \dots$



- What is ultimate fate of the energy? \rightarrow Collisionless energy transfer mechanisms?

Motivation

- Transfer vs Transport

$$\frac{3}{2} n \frac{\partial T_\alpha}{\partial t} + \underbrace{\nabla \cdot \mathcal{Q}_\alpha}_{\text{Transport}} = \underbrace{\langle \tilde{E} \cdot \tilde{J}_\alpha \rangle}_{\text{collisionless transfer}} \mp \underbrace{n \frac{m_e}{m_i} (T_e - T_i)}_{\text{Collisional transfer}} + \dots \text{heat balance; } \alpha = e, i$$

→ \mathcal{Q} heat flux, energy loss by turbulent transport

→ $\langle \tilde{E} \cdot \tilde{J}_\alpha \rangle$ { turbulent heating for single species: **electron cooling, ion heating**
 electron and ion collisionless energy transfer, local energy "source" or "sink"

→ $\mp n \frac{m_e}{m_i} (T_e - T_i)$: electron and ion heat transfer by collisions

★ ITER: low collisionality, electron heated plasma

- Collisionless energy transfer likely dominant!!

- Issues with turbulent heating: $\langle \tilde{E} \cdot \tilde{J} \rangle = \sum_{\alpha=e,i} \langle \tilde{E} \cdot \tilde{J}_\alpha \rangle$

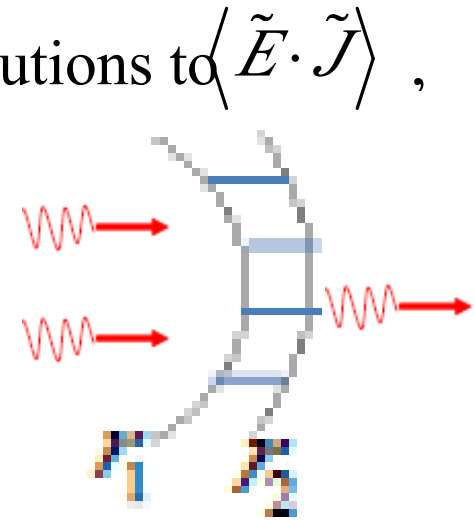
★ **Classic Question:**

- **Is the net heating zero?** (Manheimer 77)
- If periodic boundary condition, no boundary contributions to $\langle \tilde{E} \cdot \tilde{J} \rangle$,
so $\langle \tilde{E} \cdot \tilde{J} \rangle = 0$

But

$$\int dr \langle \tilde{E} \cdot \tilde{J} \rangle = \underbrace{-\tilde{\varphi} \tilde{J}_r}_{\text{Surface term survives!}} \Big|_{r_1}^{r_2} + \int dr (\nabla \cdot \tilde{J} \tilde{\varphi}) \neq 0$$

Surface term survives! → Net heating



→ Boundary effect in a finite annular region

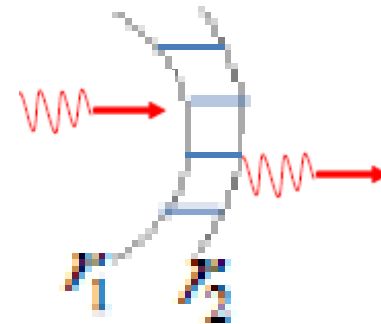
★ Point : $\langle \tilde{E} \cdot \tilde{J} \rangle \neq 0$

• *Another perspective: wave energy theorem*

$$\frac{\partial W}{\partial t} + \vec{\nabla} \cdot \vec{S} + \langle \tilde{E} \cdot \tilde{J} \rangle = 0$$

$W \equiv$ Wave energy density

$S \equiv$ wave energy density flux



• *At steady state*

$$\int_{r_1}^{r_2} dr \langle \tilde{E} \cdot \tilde{J} \rangle = -S_r \Big|_{r_1}^{r_2}$$

$$S_r = V_{gr,r} \epsilon_\omega = -2 \frac{\rho_s^2 k_r k_\theta \epsilon_\omega V_* \epsilon_\omega}{(1 + k_\perp^2 \rho_s^2)^2}$$

➡ wave energy flux differential

➡ *net heating*

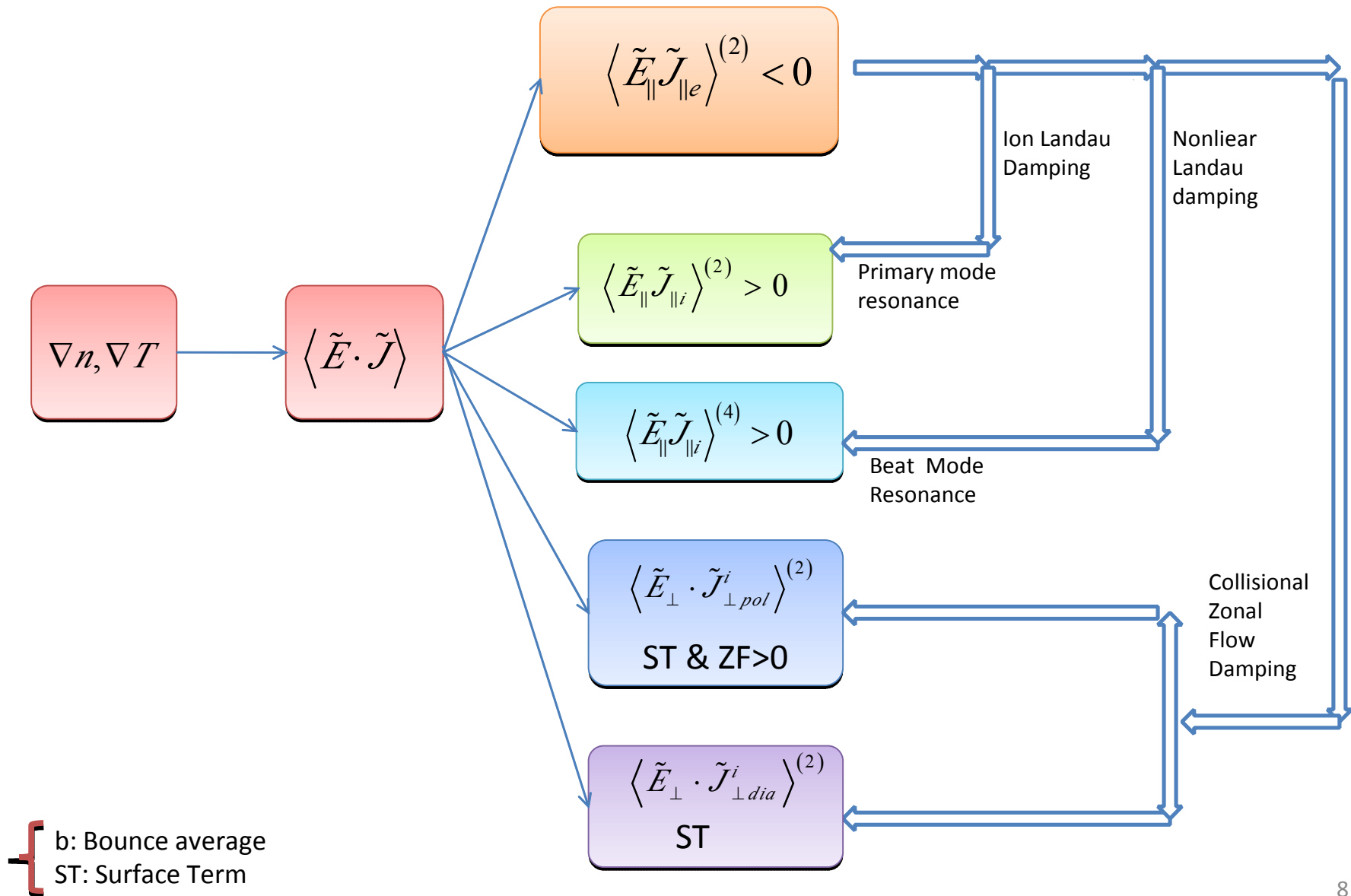
$$\langle \tilde{V}_r \tilde{V}_\theta \rangle = \sum -k_r k_\theta |\tilde{\phi}|^2$$

— Related to Reynolds work

★ *We need reconsider both the total turbulent heating and energy transfer channels in an annular region!*

- Collisionless, inter-species energy transfer
 - Where does the net energy transfer go?
 - How is energy transferred from electrons to ions (**turbulent transfer channels**)?
 - How relate to saturation mechanisms?
 - Role of ZF in heating?
 - **ZF is important to saturation, so must enter energy transfer as well!?**
 - **Zonal flow damping by friction is an energy sink**
 - **Nonlinear wave-particle interaction (considered in future) is another possibility**

Turbulent Energy flow Channels



● **Necessary Correspondence:** Nonlinear Saturation and Energy Transfer

- Nonlinear saturation in turbulent state implies energy transfer from source $(\nabla T_e, \nabla n)$ to sink
- Schematically, saturation implies some balance condition must be satisfied

i.e. $0 = \gamma = \gamma_{\substack{\text{Linear} \\ \text{electron}}} + \gamma_{\substack{\text{Linear} \\ \text{ion}}} + \gamma_{\substack{\text{Zonal} \\ \text{Flow}}} + \gamma_{\text{NLLD}} + \dots$

>0
 <0
 <0
 <0

- Channels for electron \rightarrow ion energy transfer **must** be consistent with saturation balance

In particular:

- If zonal flows control saturation, they **must** contribute to energy transfer and dissipation.
- As zonal flows are nonlinearly generated (Reynolds stress), we should consider other nonlinear heating channels.

Quasilinear Turbulent Heating in Electron Drift Wave

(Prototype; **CTEM**- specific later)

- Calculate $\langle \tilde{E}_{\parallel} \tilde{J}_{\parallel e} \rangle^{(2)}$ in quasilinear theory

- DKE for electron

- Take non-adiabatic electron distribution function

$$\tilde{g}_k = \frac{(\omega_* - \omega)}{\omega - k_z v_z} \frac{e\tilde{\varphi}_k}{T_e} \langle f_e \rangle, \quad \omega_{*e} = \frac{k_y \rho_s c_s}{L_n}, \quad \langle f_e \rangle \text{ is Maxwellian}$$

$$\langle \tilde{E}_{\parallel} \tilde{J}_{\parallel e} \rangle^{(2)} = e \int dv v_z \tilde{E}_z \tilde{g}_k = \sum_k \pi n T_e \left| \frac{e\tilde{\varphi}_k}{T_e} \right|^2 \frac{\omega}{|k_z| V_{the}} (\omega - \omega_{*e}) \langle f_e \rangle_{\frac{\omega}{k_z}}$$

- $\omega = \frac{\omega_{*e}}{1 + k_{\perp}^2 \rho_s^2}$, $\langle \tilde{E}_{\parallel} \tilde{J}_{\parallel e} \rangle^{(2)} < 0$ **the electrons cool via inverse electron**

Landau damping

- Similarly, calculate $\langle \tilde{E}_{\parallel} \tilde{J}_{\parallel i} \rangle^{(2)}$ for ion

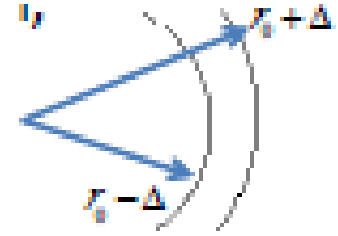
$$\langle \tilde{E}_{\parallel} \tilde{J}_{\parallel i} \rangle^{(2)} = \sum_k \pi n T_i \left| \frac{e\tilde{\varphi}_k}{T_i} \right|^2 \frac{\omega}{|k_z| V_{the}} \left(\omega + \frac{T_i}{T_e} \omega_{*e} \right) \langle f_i \rangle_{\frac{\omega}{k_z}}$$

- $\langle \tilde{E}_{\parallel} \tilde{J}_{\parallel i} \rangle^{(2)} > 0$, **the ions gain energy via ion Landau damping**

Perpendicular Current Induced Turbulent Heating

- The turbulent heating induced by ion polarization current

$$\langle \tilde{E}_\perp \cdot \tilde{J}_{\perp i}^{pol} \rangle = - \langle \vec{\nabla}_\perp \cdot (\tilde{\phi} \tilde{J}_{\perp i}^{pol}) \rangle + \langle \tilde{\phi} \vec{\nabla}_\perp \cdot \tilde{J}_{\perp i}^{pol} \rangle$$



$$\text{--- Defining an annular region } \langle \dots \rangle = \int_0^{2\pi R} dz \int_0^{2\pi} r d\theta \int_{r_0 - \Delta}^{r_0 + \Delta} (\dots) dr$$

- Perpendicular heating \longleftrightarrow Reynolds work on zonal flow

$$\langle \tilde{E}_\perp \cdot \tilde{J}_{\perp i}^{pol} \rangle = nm_i A \left(\underbrace{\langle V_\theta \rangle \langle \tilde{V}_r \tilde{V}_\theta \rangle}_{\text{wave energy flux}} \mathbb{I}_{r-\Delta}^{r+\Delta} - \underbrace{\int_{r-\Delta}^{r+\Delta} dr \langle V_\theta \rangle \frac{\partial}{\partial r} \langle \tilde{V}_r \tilde{V}_\theta \rangle}_{\text{Directly linked to zonal flow drive}} \right)$$

- wave energy flux
- Directly linked to zonal flow drive

- At steady state

$$\langle \tilde{E}_\perp \cdot \tilde{J}_{\perp i}^{pol} \rangle = \int_{r-\Delta}^{r+\Delta} dr \nu_{col} \langle V_\theta \rangle^2 > 0, \quad \longrightarrow \text{Zonal flow frictional damping is the final fate of net electron-ion energy transfer}$$

- Diamagnetic current induced turbulent heating

$$\langle \tilde{E}_\perp \cdot \tilde{J}_{\perp i}^{Dia} \rangle = -nc\tilde{\phi} \frac{B \times \nabla \tilde{p}}{B^2} \mathbb{I}_{r-\Delta}^{r+\Delta} \longrightarrow \text{Heat flux differential}$$

- other damping possible

Overview of Results

★ Estimation of the turbulent heating contributions:

Turbulent heating	analytical	Using mixing length approximation $\frac{e\tilde{\varphi}}{T_e} \sim \rho_*$ for fluctuation levels
$\left \left\langle \tilde{E} \cdot \tilde{J} \right\rangle_e^{(2)} \right $	$\sum \pi n T_e \left \frac{e\tilde{\varphi}}{T_e} \right ^2 \frac{(\omega - \omega_{*e}) \omega}{ k_z V_{the}}$	$\rho_*^2 \frac{Rq\omega_*^2}{V_{the}} F_1(k_\perp \rho_s)$
$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle_i^{(2)}$	$\sum \pi n T_i \left \frac{e\tilde{\varphi}}{T_i} \right ^2 \frac{(\omega - \omega_{*i}) \omega}{ k_z V_{thi}} \exp\left(-\frac{\omega}{k_z V_{thi}}\right)^2$	$\rho_*^2 \frac{Rq\omega_*^2}{V_{thi}} F_2(k_\perp \rho_s)$
$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle_{pol}^{(2)}$	$n m_i v_{col} \left\langle V_\theta \right\rangle^2$	$\rho_*^2 v_* \varepsilon^{3/2} m_i C_s^2 \frac{V_{thi}}{Rq}$

Basic comparison of channels

ITER Parameters R=6.2m,
a=2m,q=2

$\text{Ratio} = \frac{\langle \tilde{E} \cdot \tilde{J}_i \rangle}{\langle \tilde{E} \cdot \tilde{J}_e \rangle}$	Short wavelength
	$k_{\perp} \rho_s \sim 1$
$\langle \tilde{E} \cdot \tilde{J} \rangle_i^{(2)}$	1.56×10^{-2}
$\langle \tilde{E} \cdot \tilde{J} \rangle_{pol}^{(2)}$	$0.8 v_*$

★ Ratios of energy dissipation channels at different collisionality

■ Ion Landau Damping
■ Zonal flow friction



$$v_* = 10^{-3}, \rho^* = 10^{-3}$$



$$v_* = 10^{-2}, \rho^* = 10^{-3}$$

★ Zonal flow frictional damping can be a significant dissipation channel

★ "Collisionless drift wave" $\omega \gg v_* > 0$

Turbulent Heating \longrightarrow NL wave-particle Interaction

- ZF coupling to $o\left(\left|\frac{e\tilde{\varphi}}{T}\right|^4\right)$, so need calculate parallel heating to $o\left(\left|\frac{e\tilde{\varphi}}{T}\right|^4\right)$
- The ion diffusion equation in electron drift wave turbulence:

$$\frac{\partial \langle f_i \rangle}{\partial t} = \frac{\partial}{\partial v} D_{vv} \frac{\partial}{\partial v} \langle f_i \rangle + \frac{\partial}{\partial v} D_{vx} \frac{\partial}{\partial x} \langle f_i \rangle + \frac{\partial}{\partial x} D_{xv} \frac{\partial}{\partial v} \langle f_i \rangle + \frac{\partial}{\partial x} D_{xx} \frac{\partial}{\partial x} \langle f_i \rangle$$

Diffusion coefficient in quasilinear and nonlinear order

$$D_{vv} = D_{vv}^{(2)} + D_{vv}^{(4)} + \dots \quad D_{vx} = D_{xv} = D_{vx}^{(2)} + D_{vx}^{(4)} + \dots \quad D_{xx} = D_{xx}^{(2)} + D_{xx}^{(4)} + \dots$$

The resonant ion kinetic energy evolution :

$$\begin{aligned} \frac{\partial E_{res}}{\partial t} &= \int dv \frac{1}{2} m v^2 \frac{\partial}{\partial t} \langle f_i \rangle \\ &= - \int dv m v D_{vv} \frac{\partial}{\partial v} \langle f_i \rangle - \int dv m v D_{vx} \frac{\partial}{\partial x} \langle f_i \rangle + \frac{\partial}{\partial x} \int dv \frac{1}{2} m v^2 (D_{xv} \frac{\partial}{\partial v} \langle f_i \rangle + D_{xx} \frac{\partial}{\partial x} \langle f_i \rangle) \\ &= (\langle \tilde{E} \cdot \tilde{J} \rangle_{vv}^{(2)} + \langle \tilde{E} \cdot \tilde{J} \rangle_{vv}^{(4)} + \dots) + (\langle \tilde{E} \cdot \tilde{J} \rangle_{vx}^{(2)} + \langle \tilde{E} \cdot \tilde{J} \rangle_{vx}^{(4)} + \dots) + (\nabla \cdot q^{(2)} + \nabla \cdot q^{(4)} + \dots) \end{aligned}$$

- The nonlinear ion turbulent heating up to $o\left(\left|\frac{e\tilde{\varphi}}{T}\right|^4\right)$ \longrightarrow q is $o\left(\left|\frac{e\tilde{\varphi}}{T}\right|^4\right)$ energy flux

$$\longrightarrow \langle \tilde{E} \cdot \tilde{J} \rangle_i^{(4)} = \langle \tilde{E} \cdot \tilde{J} \rangle_{vv}^{(4)} + \langle \tilde{E} \cdot \tilde{J} \rangle_{vx}^{(4)}$$

- The fourth order nonlinear diffusion coefficient

$$D_{vv}^{(4)} = \int_{-\infty}^{\infty} \frac{\langle \tilde{F}_2(t) \tilde{F}_2(t+\tau) \rangle}{2} d\tau \rightarrow \text{perturbation theory (Dupree 68)}$$

- The particle orbit is expanded as : $x(t) = x_0 + v_0 t + \varepsilon x_1(t) + \varepsilon^2 x_2(t) + \dots$,
- The force in perturbative electric field

$$\begin{aligned}
 D_{vv}^{(4)} &= D_{v_{zx}v_{zx}}^{(4)} + D_{v_{zy}v_{zy}}^{(4)} + D_{v_{zz}v_{zz}}^{(4)} \\
 &= \sum_{k,k',-k,-k'} \left(\frac{ec}{mB} \right)^2 |\tilde{\phi}_k|^2 |\tilde{\phi}_{k'}|^2 \left(\frac{k_\theta k'_x k'_z}{\omega - k_z v_z} + \frac{k_x k'_\theta k'_z}{\omega' - k'_z v_z} \right)^2 \text{Re} \frac{i}{\omega'' - k_z'' v} \\
 &+ \sum_{k,k',-k,-k'} \left(\frac{ec}{mB} \right)^2 |\tilde{\phi}_k|^2 |\tilde{\phi}_{k'}|^2 \left(\frac{k_x k'_\theta k'_z}{\omega - k_z v_z} + \frac{k_\theta k'_x k'_z}{\omega' - k'_z v_z} \right)^2 \text{Re} \frac{i}{\omega'' - k_z'' v} \\
 &+ \sum_{k,k',-k,-k'} \left(\frac{e}{m} \right)^4 |\tilde{E}_z|^2 |\tilde{E}_{z'}|^2 \left(\frac{k_z - k'_z}{(\omega - k_z v_z)(\omega' - k'_z v_z)} \right)^2 \text{Re} \frac{i}{\omega'' - k_z'' v}
 \end{aligned}$$

Nonlinear diffusion in velocity space corrected by E×B drift motion

Nonlinear diffusion in Vlasov plasma, corrected by v_{\parallel} scattering (Dupree 68)

$\rightarrow D_{v_{zx}v_{zx}}^{(4)} \approx D_{v_{zy}v_{zy}}^{(4)} \gg D_{v_{zz}v_{zz}}^{(4)} \approx 1/\rho_*^2$ Nonlinear diffusion correction by E×B scattering is bigger than the one by parallel acceleration

- Similarly, the cross term of nonlinear diffusion coefficient

$$D_{vx}^{(4)} = \int_{-\infty}^{\infty} \frac{\langle \tilde{F}_2(t) \tilde{V}_2(t+\tau) \rangle}{2} d\tau \quad \text{and} \quad D_{v_{zz}x_{xx}}^{(4)} \approx D_{v_{zz}x_{xy}}^{(4)} \gg D_{v_{zz}x_{xz}}^{(4)} \approx 1 / \rho_*^2$$

- The nonlinear turbulent heating for ions \rightarrow due to Ion Compton Scattering

$$\begin{aligned} \langle \tilde{E} \cdot \tilde{J} \rangle_i^{(4)} &= \langle \tilde{E} \cdot \tilde{J} \rangle_{vw}^{(4)} + \langle \tilde{E} \cdot \tilde{J} \rangle_{vx}^{(4)} \\ &\approx \sum_{k, -k, k', -k'} \pi n T_i \frac{V_{thi}^3}{\omega_{ci}^2} \left| \frac{\tilde{e}\tilde{\varphi}_k}{T_i} \right|^2 \left| \frac{\tilde{e}\tilde{\varphi}_{k'}}{T_i} \right|^2 \frac{\omega'' k_z}{k_z'' |k_z''|} \left(\frac{k_\theta k'_x}{\omega - k_z v_z} + \frac{k_x k'_\theta}{\omega' - k'_z v_z} \right)^2 \left(\frac{\omega''}{k_z''} k_z + \frac{T_i}{T_e} \omega_{*e} \right) \langle f_i \rangle \mathbb{I}_{\frac{\omega''}{k}} > 0 \end{aligned}$$

Beat mode resonance : $\omega'' = \omega \pm \omega', k'' = k \pm k'$

Turbulent energy transfer : Electron \rightarrow wave $\xrightarrow{\text{NLLD}}$ Ion

- Estimate $\langle \tilde{E} \cdot \tilde{J} \rangle_i^{(4)}$ using the mixing length approximation

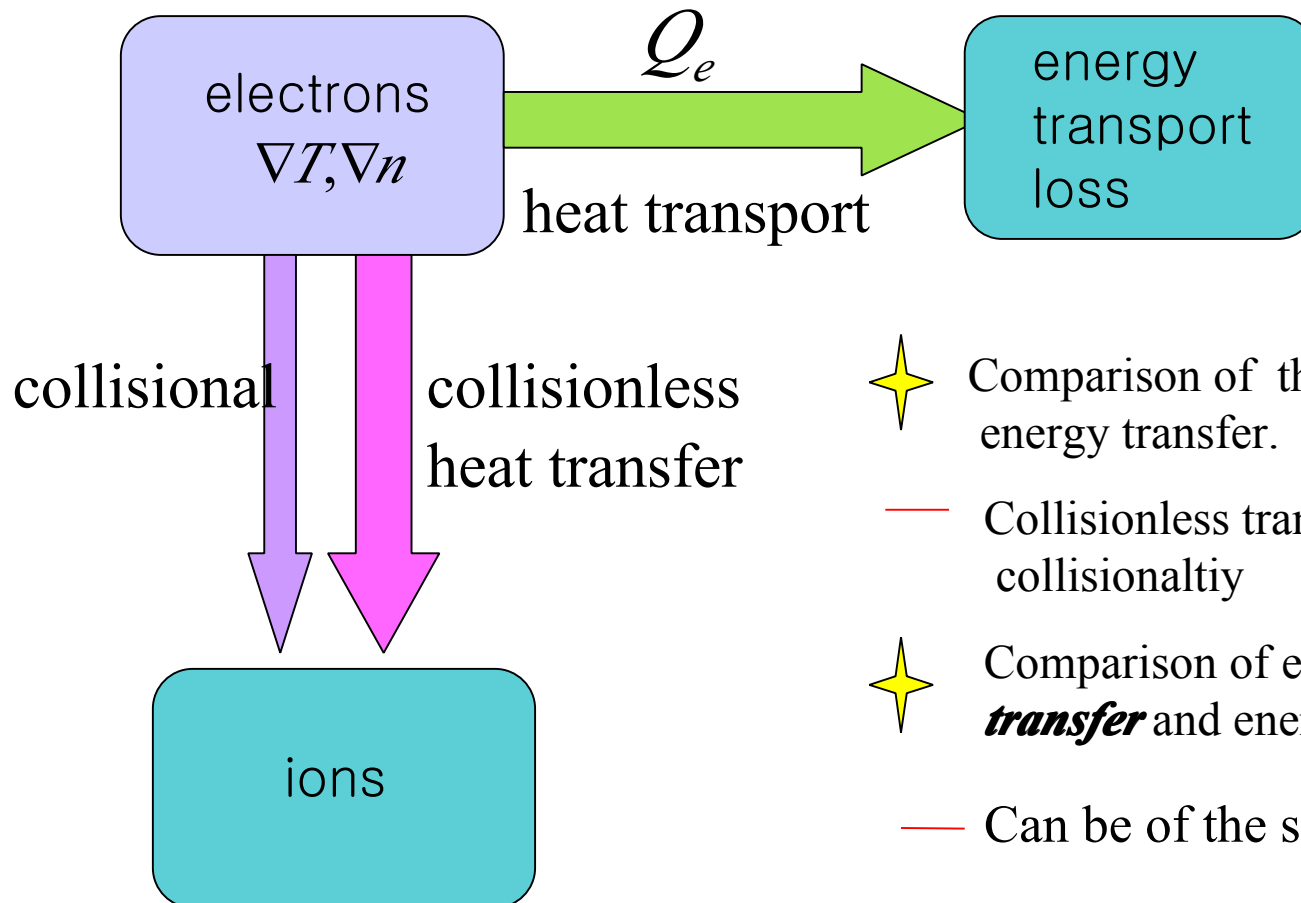
\rightarrow $\langle \tilde{E} \cdot \tilde{J} \rangle_i^{(4)} > \langle \tilde{E} \cdot \tilde{J} \rangle_i^{(2)}$ Mixing length approximation *overestimates* the turbulence intensity in the weak turbulence theory. (T.S. Hahm, 1991)

\rightarrow $\gamma_L N + \gamma_{NL} N^2 + \gamma_{ZF} N = 0$ N has to be *self-consistent* (Diamond, 2005)

Implication → Bottom Line

- Electron turbulent energy transport

$$\frac{3}{2} n \frac{\partial T_e}{\partial t} + \nabla \cdot Q_e = \langle \tilde{E} \cdot \tilde{J}_e \rangle - n \nu \frac{m_e}{m_i} (T_e - T_i) \rightarrow \text{Electron heat balance}$$



★ Comparison of the collisional and collisionless energy transfer.

— Collisionless transfer can dominate at low collisionality

★ Comparison of energy transfer in collisionless **transfer** and energy **transport** by heat flux Q

— Can be of the same order!

Collisionality

- Collisionality ν_* in ITER

– dimensionless
$$\nu_* = \frac{\varepsilon^{-3/2} R q \nu_e}{V_{the}} \longrightarrow \nu_* \sim 10^{-3}$$

- Collisionality at crossover of collisional and collisionless coupling

– Energy transfer in collision :
$$Q_i \approx \frac{n m_e \nu_e}{m_i} T_e$$

- Quasilinear trapped electron cooling in CTM

$$\left\langle \bar{\vec{E}} \cdot \bar{\vec{J}}_e \right\rangle_b^{(2)} \simeq 4\pi^{1/2} \varepsilon^{1/2} n T_e \left(\frac{R}{2a} \right)^{3/2} \rho_*^2 (\omega - \omega_*) \langle f_e \rangle \Big|_{E = \frac{\omega}{k_\theta} \frac{R T_e}{\rho_s C_s}}$$

– **At crossover** :
$$Q_i \approx \left\langle \bar{\vec{E}} \cdot \bar{\vec{J}}_e \right\rangle_b^{(2)} \longrightarrow \nu_* \sim 10^{-2}$$

★ The collisionless turbulent energy transfer then beats collisional inter-species coupling process!

★ Collisionless process will control electron-ion transfer in ITER

Transfer vs Transport

- The **transfer** and **transport** energy loss in CTEM
 - Compare the volume integral of the electron cooling to the surface integrated of the electron heat flux

$$A\tilde{Q}_e \mathbb{I}_{boundary} = \int d^3 r \langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}} \rangle$$

- The heat flux for electrons : $\tilde{Q}_e = \langle \tilde{v}_r \tilde{P}_e \rangle = -\frac{c}{B} \sum_k k_\theta \text{Im} \tilde{P}_e^{(1)} \tilde{\varphi}$

- The pressure fluctuation $\tilde{p}_e^{(1)} = \int d^3 v \frac{1}{2} m v^2 \tilde{g}_b$

$$\tilde{Q}_e = \sum 4\pi^{\frac{1}{2}} \varepsilon^{\frac{1}{2}} \left(\frac{R}{L_n} \right)^{\frac{5}{2}} \left| \frac{e\tilde{\varphi}}{T_e} \right|^2 \frac{V_{the}^2 k_\theta n T_e}{\Omega_e} (\omega - \omega_{*n}) \langle f_e \rangle \mathbb{I}_{E=\frac{\bar{\omega}_{de} R T_e}{\omega_* L_n}}$$



The ratio

$$\frac{\Delta r \langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{J}} \rangle}{\tilde{Q}_e \mathbb{I}_{boundary}} \approx 2 \frac{a}{R} \sim o(1)$$

The rate of electron energy lost by collisionless energy transfer is comparable to turbulent transport by CTEM!

Result and Discussion

- Net heating
 - Quasilinear turbulent energy transfer in drift wave
 - Nonlinear ion heating by beat wave resonance
 - Energy flux differential gives rise to the net heating → Zonal flow
- Energy transfer channels
 - Identify important energy transfer channels
 - Zonal flow frictional damping can be comparable to LD
- For low collisionality ITER plasma, collisionless energy transfer is a critical element of transport model.
 - it is same order as transport
- ★ In the future, an important task for ITER transport modelling
 - Develop a tractable representation of the *turbulent heating* and *collisionless energy transfer*

Some details for CTEM

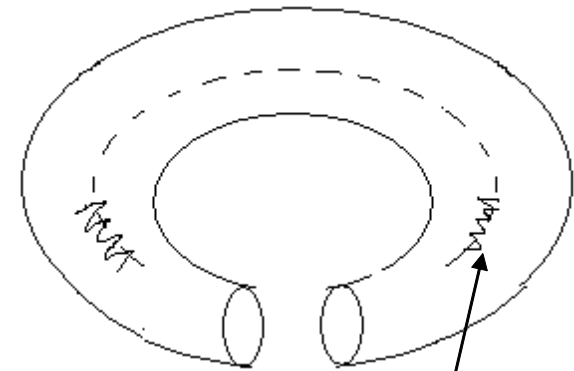
Realistic application: details of turbulent energy transfer in CTEM

- Trapped electron precession frequency:

$$\bar{\omega}_d \approx \frac{k_\theta \rho_s c_s}{R} \frac{E}{T_e}$$

- Bounce averaged kinetic equation for CTEM

$$\bar{\mathcal{G}}_{k,\omega} = -\frac{e}{T_e} \langle f_e \rangle \frac{\omega - \omega_*}{\omega - \omega_d + i\nu} \langle e^{-inq\theta} \tilde{\Phi}_{k,\omega} \rangle_b$$



Toroidal precession of trapped particles

- turbulent heating for quasilinear trapped electron

$$\langle \bar{\mathbf{E}} \cdot \bar{\mathbf{J}}_e \rangle_b^{(2)} = e \int d^3v \bar{\nu}_d \bar{\mathbf{E}} \bar{\mathcal{G}}_{k,\omega} = \sum 4\pi^{1/2} \varepsilon^{1/2} n T_e \left(\frac{R}{2a} \right)^{3/2} \rho_*^2 (\omega - \omega_*) \langle f_e \rangle \mathbb{I}_{E = \frac{\omega R T_e}{k_\theta \rho_s c_s}}$$

★ wave-trapped particle resonance: $\omega = \bar{\omega}_d$

➔ $\omega = \frac{\omega_*}{1 + k_\perp^2 \rho_s^2} \Rightarrow \langle \bar{\mathbf{E}} \cdot \bar{\mathbf{J}}_e \rangle_b^{(2)} < 0$ **Trapped electron cooling via trapped electron precession resonance**

Nonlinear trapped electron cooling

- The beat mode resonance effect for trapped electrons (L.Chen'77)
(Gang'90)
 - The 3rd order nonlinear trapped electron response function

$$\hat{h}_n^{e(3)} = -\left(\frac{nq}{2r}\right)^2 d_{\theta\theta}^e \langle e^{-inq\theta} \tilde{\varphi}_n \rangle_b + d_{rr}^e \frac{\partial^2}{\partial r^2} \langle e^{-inq\theta} \tilde{\varphi}_n \rangle_b$$

$$d_{\theta\theta}^e = \left(\frac{c}{B}\right)^2 \frac{1}{\omega} \sum_n \frac{1}{\omega'' - \omega_{de}''} \left| \frac{\partial}{\partial r} \langle e^{-in'\theta} \tilde{\varphi}_{n'} \rangle_b \right|^2 \left(\frac{\omega_e^{*'}}{\omega'} - \frac{\omega_e^*}{\omega} \right) \frac{e}{T_e} \langle f_e \rangle$$


$$d_{rr}^e = \left(\frac{c}{B}\right)^2 \frac{1}{\omega} \sum_n \frac{1}{\omega'' - \omega_{de}''} \left(\frac{nq}{r}\right)^2 \left| \langle e^{-in'\theta} \tilde{\varphi}_{n'} \rangle_b \right|^2 \left(\frac{\omega_e^{*'}}{\omega'} - \frac{\omega_e^*}{\omega} \right) \frac{e}{T_e} \langle f_e \rangle$$

- $1/\omega'' - \omega_{de}'' \rightarrow$ the nonlinear trapped electron-wave interaction

- The nonlinear turbulent heating for trapped electrons

$$\left\langle \bar{\tilde{E}} \cdot \bar{\tilde{J}}_e \right\rangle_b^{(4)} = e \int d^3 v \bar{V}_d \bar{\tilde{E}}_d \hat{h}_n^{e(3)}$$

$$\rightarrow -\pi^2 \left(\frac{\varepsilon}{2}\right)^{\frac{1}{2}} \sum_{n,n'} n_0 T_e \left| \frac{e \bar{\varphi}_{n,\omega}}{T_e} \right|^2 \left| \frac{e \bar{\varphi}_{n',\omega'}}{T_e} \right|^2 \frac{V_{the}^4 \rho_s^4}{\Omega_e^2 |k_\theta''|} (k_\theta^2 - k_\theta'^2)^2 (k_\theta^2 k_r'^2 + k_\theta'^2 k_r^2) \left(\frac{\omega'' RT_e}{k'' \rho_s c_s} \right)^{\frac{3}{2}} \langle f_e \rangle \Big|_{E=\frac{\omega'' RT_e}{k'' \rho_s c_s}} < 0$$

 **In CTEM, trapped electron can have quasilinear and nonlinear cooling. Nonlinear cooling can be significant.**

Related experimental phenomenon

- Electron temperature profile "**stiffness**"
 - temperature profile react weakly to changes of auxiliary heating deposition

C.C . Petty 94'

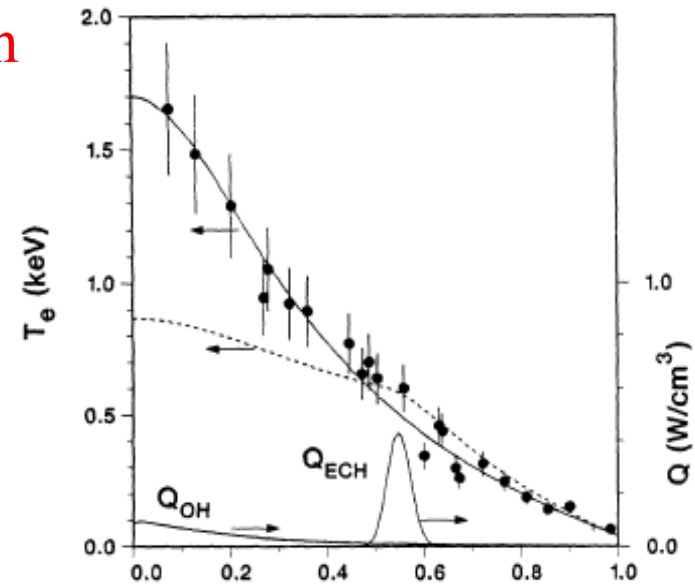
Possible dynamical cause

- Heat pinch : **inward flow**
- Nondiffusive term in heat flux**
- Electron- ion energy transfer in the core: "**Sink**"

- They are two **different and independent effects** . *which one is more efficient? Both must be examined..*

$$\frac{\partial T_e}{\partial t} + \nabla \cdot Q_e = \langle \tilde{E} \cdot \tilde{J}_e \rangle$$

$$Q_e = -\chi_e n_e \nabla T_e + \underbrace{V_e n_e T_e}_{\text{heat pinch}}$$



Experimental electron temperature profile measured in DIII-D tokamak.

(Weiland ' 89), (L. Wang '2011)

Conclusion

- ITER plasma is low collisionality, electron heated plasma
- Turbulent heating and Turbulent energy transfer channels
 - Net heating occurs!
 - Zonal flow (& frictional damping) can be significant energy transfer channel (and sink) for CTEM turbulence
- Collisionless energy transfer likely dominant in energy coupling and a critical element in transport analysis
 - Does this process excite ITG?
- Future work extend to :
 - Realistic model: CTEM
 - Representation for energy transfer in ITER modelling
 - Future experiment :electron temperature profile "stiffness"
 - $o\left(\left|\frac{e\tilde{\phi}}{T}\right|\right)^4$ contributions to Residual stress, in momentum flux